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A SIMPLE, APPROXIMATE METHOD FOR ANALYSIS OF KERR-NEWMAN BLACK HOLE DYNAMICS AND THERMODYNAMICS

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Abstract

In this work we present a simple, approximate method for analysis of the basic dynamical and thermodynamical characteristics of Kerr-Newman black hole. Instead of the complete dynamics of the black hole self-interaction we consider only such stable (stationary) dynamical situations determined by condition that black hole (outer) horizon circumference holds the integer number of the reduced Compton wave lengths corresponding to mass spectrum of a small quantum system (representing quant of the black hole self-interaction). Then, we show that Kerr-Newman black hole entropy represents simply the quotient of the sum of static part and rotation part of mass of black hole on the one hand and ground mass of small quantum system on the other hand. Also we show that Kerr-Newman black hole temperature represents the negative value of the classical potential energy of gravitational interaction between a part of black hole with reduced mass and small quantum system in the ground mass quantum state. Finally, we suggest a bosonic great canonical distribution of the statistical ensemble of given small quantum systems in the thermodynamical equilibrium with (macroscopic) black hole as thermal reservoir. We suggest that, practically, only ground mass quantum state is significantly degenerate while all other, excited mass quantum states are non-degenerate. Kerr-Newman black hole entropy is practically equivalent to the ground mass quantum state degeneration. Given statistical distribution admits a rough (qualitative) but simple modeling of Hawking radiation of the black hole too.

1 Introduction

In this work, generalizing our previous result on Schwarzschild and Kerr-Newman black hole [1]-[3], we shall present a simple, approximate method for analysis of the basic dynamical and thermodynamical characteristics (Bekenstein-Hawking entropy and Hawking temperature) of Kerr-Newman black hole. Instead of the complete dynamics of Kerr-Newman black hole self-interaction we shall

consider only such stable (stationary) dynamical situations determined by condition that Kerr-Newman black hole (outer) horizon circumference holds the integer number of the reduced Compton wave lengths corresponding to mass spectrum of a small quantum system (representing quant of Kerr-Newman black hole self-interaction). (Obviously it is conceptually analogous to Bohr quantization postulate interpreted by de Broglie relation in Old, Bohr-Sommerfeld, quantum theory. Also, it can be pointed out that our formalism is not theoretically dubious, since, at it is not hard to see, it can represent an extreme simplification of a more accurate, e.g. Copeland-Lahiri [4], string formalism for the black hole description.) Then, we shall show that Kerr-Newman black hole entropy represents the quotient of the sum of static (Schwarzschild) part and rotation part of mass of Kerr-Newman black hole on the one hand and ground mass of small quantum system on the other hand. Also we shall show that black hole temperature represents the negative value of the classical potential energy of gravitational interaction between a part of black hole with reduced mass and small quantum system in the ground mass quantum state. Finally, we shall suggest a bosonic great canonical distribution of the statistical ensemble of given small quantum systems in the thermodynamical equilibrium with (macroscopic) Kerr-Newman black hole as thermal reservoir. We shall suggest that, practically, only ground mass quantum state is significantly degenerate while all other, excited mass quantum states are non-degenerate. Kerr-Newman black hole entropy is practically equivalent to the ground mass quantum state degeneration. Given statistical distribution admits a rough (qualitative) but simple modeling of Hawking radiation of the black hole too. In many aspects this modeling is very close to Parikh and Wilczek modeling of Hawking radiation as tunneling [5].

2 Theory

As it is well-known [6] outer horizon radius of Kerr-Newman black hole is given by expression

$$R = M + (M^2 - a^2 - Q^2)^{\frac{1}{2}} \tag{1}$$

where M represents the black hole mass, $a = \frac{J}{M}$ where J represents the black hole angular momentum, while Q represents the black hole electric charge. It implies

$$M = \frac{R}{2} + \frac{1}{2}\frac{a^2}{R} + \frac{1}{2}\frac{Q^2}{R} = M_s + M_r + M_c. \tag{2}$$

First part of M, $M_s = \frac{R}{2}$, can be considered as an effective black hole mass corresponding to a fictitious Schwarzschild black hole with horizon radius R. In fact M_s can be considered as the mass corresponding to static part of the gravitational field of Kerr-Newman black hole.

Second part of M, $M_r = \frac{1}{2}\frac{a^2}{R}$, represents classically the mass, i.e. rotation kinetic energy corresponding to angular momentum $J = aM$ and radius R.

We can introduce the following

$$M_g = M_s + M_r = \frac{R}{2} + \frac{1}{2}\frac{a^2}{R} = \frac{R^2 + a^2}{2R} \tag{3}$$

$$R_g = 2M_g. \tag{4}$$

Here M_g can be considered as an effective mass corresponding to total gravitational mass representing sum of the static and rotation mass, while R_g can be considered as horizon radius of a fictitious Schwarzschild black hole with mass M_g.

Third part of M (2), $\frac{1}{2}\frac{Q^2}{R}$, can be considered as an effective mass, i.e. potential energy of the electrostatic repulsion of the homogeneously charged thin shell with electrical charge Q and radius R.

Finally, we can define

$$M_{red} = (M^2 - a^2 - Q^2)^{\frac{1}{2}} = M(1 - \frac{a^2 + Q^2}{M^2})^{\frac{1}{2}} \tag{5}$$

which can be considered as an effective, reduced black hole mass obtained by diminishing of the real black hole mass M by means of, classically speaking, rotation ("centrifugal force") and electrostatic repulsion.

Suppose now that, for "macroscopic" (with mass many time larger than Planck mass, i.e. 1) Kerr-Newman black hole, at horizon surface there is some small (with "microscopic" masses, i.e. masses smaller than Planck mass, i.e. 1) quantum system. It can be supposed that given small quantum system at black hole horizon represents the quant of the self-interaction of black hole, or, quant of the interaction between formally separated black hole and its fields.

Further, for a "macroscopic" Kerr-Newman black hole, instead of the complete dynamics of its self-interaction, only stable (stationary) dynamical situations will be considered. Given stability will be determined by the following condition

$$m_n R = n\frac{1}{2\pi}, \quad \text{for} \quad m_n \ll M \quad \text{and} \quad n = 1, 2, ... \tag{6}$$

where m_n for $m_n \ll M$ and $n = 1, 2, ...$ represent the mass (energy) spectrum of given small quantum system. It corresponds to expression

$$2\pi R = n\frac{1}{m_n} = n\lambda_{rn} \quad \text{for} \quad m_n \ll M \quad \text{and} \quad n = 1, 2, ... \tag{7}$$

were $2\pi R$ represents the circumference of Kerr-Newman black hole outer horizon while

$$\lambda_{rn} = \frac{1}{m_n} \tag{8}$$

represents n-th reduced Compton wavelength of mentioned small quantum system with mass m_n for $n = 1, 2, ...$. Expression (7) simply means that circumference of Kerr-Newman black hole outer horizon holds exactly n corresponding n-th reduced Compton wave lengths of given small quantum system with mass m_n captured at black hole horizon surface, for $n = 1, 2, ...$. Obviously, it is essentially analogous to well-known Bohr's angular momentum quantization postulate interpreted via de Broglie relation. (Moreover, in more accurate quantum mechanical analysis Bohr-de Broglie standing waves turn out in Schrdinger stationary quantum states, while our reduced Compton waves turn out in quantized small oscillations of Copeland-Lahiri circular (string) loop [4].) However, there is a principal difference with respect to Bohr's atomic model. Namely, in Bohr's atomic model different quantum numbers $n = 1, 2, ...,$ correspond to different circular orbits (with circumferences proportional to $n^2 = 1^2, 2^2,)$. Here any quantum number $n = 1, 2,$ corresponds to the same circular orbit (with circumference $2\pi R$).

According to (6) and (1) it follows

$$m_n = n\frac{1}{2\pi R} = n\frac{1}{2\pi(M + (M^2 - a^2 - Q^2)^{\frac{1}{2}})} \equiv nm_1 \quad \text{for} \quad m_n \ll M \quad \text{and} \quad n = 1, 2, ... \tag{9}$$

where

$$m_1 = \frac{1}{2\pi R} = \frac{1}{2\pi(M + (M^2 - a^2 - Q^2)^{\frac{1}{2}})} \tag{10}$$

represents the ground mass of small quantum system. Obviously, m_1 depends of M so that m_1 decreases when M increases and vice versa. For a "macroscopic" black hole, i.e. for $M \gg 1$ it follows $m_1 \ll 1 \ll M$.

Now, it is not hard to see that, according to (3), quotient of M_g and m_1 represents well-known Bekenstein-Hawking entropy of Kerr-Newman black hole, i.e.

$$S = \frac{M_g}{m_1} = \pi(R^2 + a^2) = \frac{A}{4}, \tag{11}$$

where, according to Bekenstein supposition, $A = 4S$ represents the black hole surface area. Obviously, it represents an interesting dynamical interpretation of Kerr-Newman black hole entropy whose statistical meaning will be discussed later.

Further, according to (3)-(5), (10), define

$$V = -\frac{M_{red} m_1}{R_g} = -\frac{(M^2 - a^2 - Q^2)^{\frac{1}{2}}}{2\pi(R^2 + a^2)} \tag{12}$$

that can be considered as the classical potential of the gravitational interaction between effective black hole part with mass M_{red} and small quantum system in the ground mass state m_1 at distance R_g.

Now, it can be observed that

$$T = -V = \frac{M_{red} m_1}{R_g} = \frac{(M^2 - a^2 - Q^2)^{\frac{1}{2}}}{2\pi(R^2 + a^2)} \tag{13}$$

represents well-known Hawking temperature of Kerr-Newman black hole. It represents an interesting dynamical interpretation of Kerr-Newman black hole temperature.

Thus Kerr-Newman black hole entropy (11) and temperature (13) are interpreted phenomenologically dynamically in a simple, quasi-classical way. Also, it can be observed that for a Schwarzschild black hole, representing an especial case of Kerr-Newman black hole for $a = 0$ and $Q = 0$, it becomes satisfied $R = 2M$, $M_s = M_g = M_{red} = M$. It implies $S = \frac{M}{m_1}$ and $T = -V = -\frac{M m_1}{R}$ representing, intuitively, very clear and simply, "obvious", quasi-classical interpretation of Schwarzschild black hole entropy (as quotient of the black hole mass and small quantum system ground mass) and temperature (as negative classical potential energy of the gravitational interaction between black hole and small quantum system in mass ground state). Vice versa, clearness and simplicity, i.e. "obviousness", of Kerr-Newman black hole entropy (11) and temperature (13) follow from fact that they represent simplest generalization of previously interpreted Schwarzschild black hole entropy and temperature.

3 Statistical meaning of Kerr-Newman black hole entropy

However, we shall give a deeper, statistical interpretation of Kerr-Newman black hole entropy.

Suppose that small quantum system interacting with (macroscopic) Kerr-Newman black hole as a thermal reservoir does a bosonic great canonical ensemble in thermodynamical equilibrium,

with mass spectrum m_n for $n = 1, 2,$ (9), temperature T (13) and chemical potential μ whose value will be determined later.

Then, as it is well-known and according to (9), statistically averaged number of the small quantum systems with mass m_n, N_n, for $n = 1, 2, ,$ is given by expression

$$N_n = g_n \frac{1}{\exp[\frac{m_n - \mu}{T}] - 1} = g_n \frac{1}{\exp[\frac{nm_1 - \mu}{T}] - 1} \qquad \text{for} \qquad n = 1, 2, ... \tag{14}$$

where g_n represents the degeneracy of the quantum state corresponding to m_n for n=1, 2,....

Also, as it is well-known too, partial entropy in the quantum state corresponding to m_n for $n = 1, 2, ,$ is given by expression

$$S_n = g_n \ln[1 + \frac{N_n}{g_n}] + N_n \ln[1 + \frac{g_n}{N_n}] \qquad \text{for} \qquad n = 1, 2, ... \tag{15}$$

where g_n represents the degeneracy of the quantum state corresponding to m_n for $n = 1, 2,$.

We shall suppose

$$g_n \simeq 1 \qquad \text{for} \qquad n \gg 1 \tag{16}$$

which, according to (14), (15) implies

$$N_n \ll 1 \qquad \text{for} \qquad n \gg 1 \tag{17}$$

and

$$S_n \simeq N_n \ll 1 \qquad \text{for} \qquad n \gg 1. \tag{18}$$

Also, we shall suppose

$$g_1 = N_1. \tag{19}$$

It, according (10)-(13), implies the following value of the chemical potential

$$\mu = m_1 - T \ln 2 = m_1(1 - \frac{T}{m_1} \ln 2) = m_1(1 - \frac{1 - \frac{M}{R}}{1 + \frac{a^2}{M^2}} \ln 2) \tag{20}$$

Intuitive explanation of the suppositions (16), (19) is very simple. Ground mass state corresponding to m1, (energetically) closest to (outer) horizon, maximally exposed to gravitational influence, is maximally degenerate. Highly excited quantum states corresponding to m_n for $n \gg 1$, (energetically) very distant from horizon, are not so strongly exposed to gravitational influence and are almost non-degenerate.

It can be observed that here we have a situation in some degree similar to Bose condensation. Small quantum systems occupy maximally, maximally degenerate ground mass state, in respect to other, practically non-degenerate, mass states even if, according to (19), $\frac{N_1}{g_1}$ does not tend toward infinity but toward 1.

According to (15)-(19) it follows

$$S_1 = 2 \ln 2 N_1 \simeq 1.39 N_1 \sim \; N_1 \qquad \text{for} \qquad n = 1. \tag{21}$$

It implies the following expression for usually statistically defined total entropy S

$$S = \sum_{n=1} S_n \simeq S_1 \simeq 1.39 N_1 \sim N_1 \tag{22}$$

and equivalence of (22) and (11) implies

$$N_1 \simeq \frac{1}{1.39}\frac{M_g}{m_1} \simeq 0.72\frac{M_g}{m_1} \sim \frac{M_g}{m_1}. \tag{23}$$

Then, statistically averaged total number of the small quantum systems N is given by expression

$$N = \sum_{n=1} N_n \simeq N_1 \simeq \frac{1}{1.39}\frac{M_g}{m_1} \simeq 0.72\frac{M_g}{m_1} \sim \frac{M_g}{m_1}. \tag{24}$$

and statistically averaged black hole gravitational mass of the ensemble $< M_g >$ is given by expression

$$< M_g >= \sum_{n=1} N_n m_n \simeq N_1 m_1 \simeq \frac{1}{1.39} M_g \simeq 0.72 M_g \sim M_g \tag{25}$$

which corresponds approximately to black hole gravitational mass M_g.

In this way we founded statistically in a satisfactory approximation all previously discussed basic thermodynamical characteristics of Kerr-Newman black hole. In other words, suggested statistics yields results in a satisfactory agreement with previous thermodynamical predictions.

However, it can be observed that supposition (16) cannot be determined by condition (19) or some other statistical or thermodynamical expression. For this reason we shall simply suppose the following form of mass (energy) degeneracy in the general case

$$g_n = (N_1 - 1)\exp[-\frac{m_n - m_1}{T}] + 1 \qquad \text{for} \qquad n = 1, 2, ... \tag{26}$$

that, for $n = 1$, is equivalent to (19), while, for $n \gg 1$, is equivalent to (16) .

4 Rough, qualitative description of black hole radiation

As it has been shown previously practically all small quantum systems from the statistical ensemble occupy ground mass quantum state. For this reason transitions (jumps) from higher into lower, especially ground, mass quantum state cannot be primary cause of black hole Hawking radiation in our simple model. In this way in our model it must be supposed that there are some additional, subtle dynamical processes, corresponding Hawking near horizon particle-antiparticle creation, which cause black hole radiation and mass decrease, on the one hand. On the other hand, given subtle dynamical processes must be presented in our simple, approximate model only roughly, phenomenologically. It can be done in a way very close to Parikh and Wilczek model of Hawking radiation as tunneling [5], or, in further conceptual analogy as a nuclear alpha decay. [5], or, in further conceptual analogy as a nuclear alpha decay.

Suppose that one, arbitrary, of S small quantum systems in ground mass quantum state interacts dynamically with other $S - 1$ small quantum systems in ground mass quantum state similarly as one alpha particle with other alpha particles in the alpha radioactive atomic nucleus. Then, like to the model of alpha decay as quantum tunneling, given interaction can be presented as propagation of one small quantum system in the potential barrier (determined by black hole and other small quantum systems) including possibility of the tunneling, i.e. small quantum system decay.

Suppose that given individual decay occurs statistically during some time interval Δt_1 and that energy of decayed small quantum system turns out in black hole radiation. Then, according to Heisenberg energy-time uncertainty relation it follows

$$\Delta t_1 \simeq \frac{1}{\Delta m_1} \tag{27}$$

where Δm_1 represents uncertainty of the mass in the ground mass quantum state corresponding to small quantum system mass m_1.

Suppose that ground mass level is sharply defined, i.e. that

$$\Delta m_1 \ll m_1 \tag{28}$$

or, for example,

$$\Delta m_1 = \frac{1}{100} m_1. \tag{29}$$

Now, total time interval for black hole complete evaporation can be roughly presented by expression

$$\Delta t_{tot} \simeq S \Delta t_1. \tag{30}$$

In the simplest case, i.e. for Schwarzschild black hole as an especial limit of Kerr-Newman black hole, (30), as it is not hard to see according to or, according to (10), (11), (27), (29),

$$\Delta t_{tot} \simeq 1600\pi^2 M^3 = 5027\pi M^3. \tag{31}$$

It is very close to Hawking time interval for total evaporation of the black hole

$$\Delta t_{tot} = 5120\pi M^3. \tag{32}$$

In this way we demonstrated that our model, very close to Parikh and Wilczek modeling of Hawking radiation as tunneling, is able to describe roughly (qualitatively) and phenomenologically, but non-trivially, black hole radiation and evaporation too.

5 Conclusion

In conclusion we shall repeat and point out the following. In this work we presented a simple, approximate method for analysis of the basic dynamical and thermodynamical characteristics (Bekenstein-Hawking entropy, and Hawking temperature) of Kerr-Newman black hole. Instead of the complete dynamics of the black hole self-interaction we considered only such stable (stationary) dynamical situations determined by condition that black hole (outer) horizon circumference holds the integer number of the reduced Compton wave lengths corresponding to mass spectrum of a small quantum system (representing quant of the black hole self-interaction). (Obviously it is conceptually analogous to Bohr quantization postulate interpreted by de Broglie relation in Old, Bohr-Sommerfeld, quantum theory. Also, it can be pointed out that our formalism is not theoretically dubious, since, at it is not hard to see, it can represent an extreme simplification of a more accurate, e.g. Copeland-Lahiri , string formalism for the black hole description.) Then, we showed that Kerr-Newman black hole entropy represents the quotient of the sum of Schwarzschild part and rotation part of mass of black hole on the one hand and ground mass of small quantum

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